

**Looking ahead. The end of the course is near.**

<https://dfdeboer.github.io/S20/304/304S20DN.HTM>

**Wednesday 4/29**

After the next class meeting the final exam (take-home style) will be made available for download.

<https://dordt.instructure.com/courses/2662877#TF>

No new labs. Prof d<sub>DB</sub> will be available in the lab for consultation. If nobody lands on the Zoom page between 1:50 and 2:10 PM Prof. d<sub>DB</sub> will close down the Zoom meeting and go home.

**Friday 5/01**

Normal class. Topics d<sub>DB</sub> hopes to talk about include security of IoT systems, message brokering services.

**Monday, 5/04**

Last day of class.

*We will review and celebrate*

Please dress in your favorite clothing that you would actually wear to a face-to-face class.

Please bring some snacks to class. Prof d<sub>DB</sub> is planning on 2 pounds of Oreos and 2 liters of Mountain Dew.

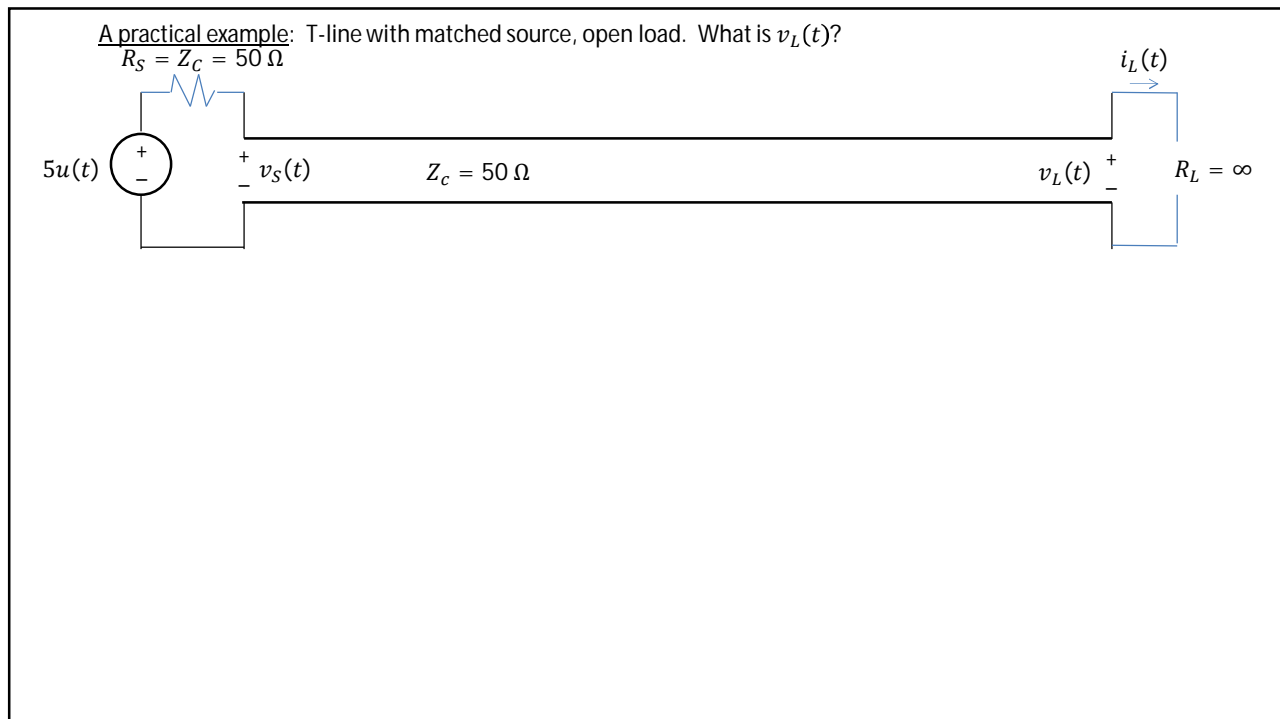
(And he is not planning to share!)

Prepare to leave your video camera on for most of the class period.

**Wednesday, 5/06**

Turn in the final exam by 11:59 PM. Class done.

1



2

A practical example: T-line with matched source, open load. What is  $v_L(t)$ ?

$R_S = Z_C = 50 \Omega$

Initial observations:

- Open load means the voltage there will go high, create a reflection.
- Matched source means the reflection will be absorbed in  $R_S$

3

A practical example: T-line with matched source, open load. What is  $v_L(t)$ ?

$R_S = 50 \Omega$

Initial observations:

- Open load means the voltage there will go high, create a reflection.
- Matched source means the reflection will be absorbed in  $R_S$

The EGR 220-style solution finds the "asymptotic" solution.

- The is the solution after all reflections have subsided.
- The t-line model parameters ( $L, C, R, G$  or  $Z_C$  and  $\gamma_C$ ) are not needed or used.
- The transmission line is treated simply as a part of two nodes, ① and ②.

This is just a voltage divider problem

$$v_L = 5u(t) \frac{R_L}{R_L + R_S} \text{ and as } t \rightarrow \infty \text{ } v_L = (5 \text{ V}) \frac{\infty}{\infty + 50 \Omega} = 5 \text{ V}$$

4

A practical example: T-line with matched source, open load. What is  $v_L(t)$ ?

Calculations: First find the reflection coefficients.

$$\Gamma_S = \frac{Z_S - Z_c}{Z_S + Z_c} = \frac{50 - 50}{50 + 50} = 0$$

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{\infty - 50}{\infty + 50} = +1$$

5

A practical example: T-line with matched source, open load

At  $t = 0^-$  (and for all  $t < 0$ ) Assume that problem starts with everything discharged,  $v_{standing} = 0$

$$\Gamma_S = \frac{Z_S - Z_c}{Z_S + Z_c} = \frac{50 - 50}{50 + 50} = 0$$

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6

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At  $t = 0^-$  (and for all  $t < 0$ ) Assume that problem starts with everything discharged,  $v_{standing} = 0$

At  $t = 0^+$  The unit step has happened. The open-circuit source voltage has stepped up to 5 V. Current will be flowing to start charging the t-line. This causes a voltage drop across  $R_S$ . The current flowing into the t-line will cause a voltage wave front to begin propagating toward the load. Finding the amount of this voltage wave front is a standard voltage-divider calculation:

Not the same as the EGR 220 asymptotic calculation. This calculation relies on t-line parameter  $Z_C$ . Initially, the t-line is the only Load on the source!

$$v_S(0^+) = (V_{OC}) \left( \frac{Z_C}{R_S + Z_C} \right) = (5 \text{ V}) \frac{50 \Omega}{50 \Omega + 50 \Omega} = 2.5 \text{ V}$$

An incident 2.5 V wave front has been launched down the t-line toward the load. In other words,  $v_o = 2.5 \text{ V}$   
 There is an associated charging current,  $i_o = (2.5 \text{ V}) / (50 \Omega) = 50 \text{ mA}$

7

A practical example: T-line with matched source, open load

$\Gamma_S = \frac{Z_S - Z_C}{Z_S + Z_C} = \frac{50 - 50}{50 + 50} = 0$ 
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$$v_{total} = v_{standing} + v_{incident} + v_{reflected} = v_{standing} + v_o + v_o \Gamma$$

At  $t = 0^-$  the T-line is fully discharged. Thus  $v_{standing} = 0$ . At  $t = 0^+$   $v_S = 5 \frac{Z_C}{R_S + Z_C} = 2.5 \text{ V}$ . Wave starts propagating.  
 Above: The previous results are summarized blue line.

8

A practical example: T-line with matched source, open load

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$$T_0 = l/V_P$$

9

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**At  $t = T_0$**  wave arrives at the load end.  $v_L = v_{standing} + v_{incident} + v_{reflected} = 0 + 2.5 + 2.5(+1) = 5$  V.

10

A practical example: T-line with matched source, open load

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Assume it takes  $T_0$  seconds for a wave to traverse the transmission line one time in one direction, say from S to L.

At  $t = T_0$  wave arrives at the load end.  $v_L = v_{standing} + v_{incident} + v_{reflected} = 0 + 2.5 + 2.5(+1) = 5$  V.

A +2.5 V wave front reflects back toward the source.

11

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At  $t = T_0$  wave arrives at the load end.  $v_L = v_{standing} + v_{incident} + v_{reflected} = 0 + 2.5 + 2.5(+1) = 5$  V.

A +2.5 V wave front reflects back toward the source.

12

**A practical example: T-line with matched source, open load**

$R_S = 50 \Omega$   
 $Z_C = 50 \Omega$   
 $R_L = \infty$

$$\Gamma_S = \frac{Z_S - Z_C}{Z_S + Z_C} = \frac{50 - 50}{50 + 50} = 0$$

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{\infty - 50}{\infty + 50} = +1$$

$$v_{total} = v_{standing} + v_{incident} + v_{reflected} = v_{standing} + v_o + v_o\Gamma$$

At  $t = 0^-$  the T-line is fully discharged. Thus  $v_{standing} = 0$ . At  $t = 0^+$   $v_S = 5 \frac{Z_C}{R_S + Z_C} = 2.5$  V. Wave starts propagating.  
 Assume it takes  $T_0$  seconds for a wave to traverse the transmission line one time in one direction, say from S to L.  
 At  $t = T_0$  wave arrives at the load end.  $v_L = v_{standing} + v_{incident} + v_{reflected} = 0 + 2.5 + 2.5(+1) = 5$  V.  
 A +2.5 V wave front reflects back toward the source. After  $T_0$  another seconds it becomes incident on the source.

**At  $t = 2T_0$**   
 An important change of perspective. Now looking at the problem from the perspective of the source, not the loa.:  
 The wave that was a *reflection from the load* (reflection: bouncing out of) is now considered an *incident wave* upon the source! (Incident: going into)

13

**A practical example: T-line with matched source, open load**

$R_S = 50 \Omega$   
 $Z_C = 50 \Omega$   
 $R_L = \infty$

$$\Gamma_S = \frac{Z_S - Z_C}{Z_S + Z_C} = \frac{50 - 50}{50 + 50} = 0$$

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{\infty - 50}{\infty + 50} = +1$$

$$v_{total} = v_{standing} + v_{incident} + v_{reflected} = v_{standing} + v_o + v_o\Gamma$$

At  $t = 0^-$  the T-line is fully discharged. Thus  $v_{standing} = 0$ . At  $t = 0^+$   $v_S = 5 \frac{Z_C}{R_S + Z_C} = 2.5$  V. Wave starts propagating.  
 Assume it takes  $T_0$  seconds for a wave to traverse the transmission line one time in one direction, say from S to L.  
 At  $t = T_0$  wave arrives at the load end.  $v_L = v_{standing} + v_{incident} + v_{reflected} = 0 + 2.5 + 2.5(+1) = 5$  V.  
 A +2.5 V wave front reflects back toward the source. After  $T_0$  another seconds it becomes incident on the source.

**At  $t = 2T_0$**   
 An important change of perspective. Now looking at the problem from the perspective of the source, not the loa.:  
 The wave that was a *reflection from the load* (reflection: bouncing out of) is now considered an *incident wave* upon the source! (Incident: going into)  
 The former reflection becomes incident on the source.  $v_S = v_{standing} + v_{incident} + v_{reflected} = 2.5 + 2.5 + 2.5(0) = 5$  V.

14

**A practical example: T-line with matched source, open load**

$R_S = 50 \Omega$   
 $Z_C = 50 \Omega$   
 $R_L = \infty$

$\Gamma_S = \frac{Z_S - Z_C}{Z_S + Z_C} = \frac{50 - 50}{50 + 50} = 0$   
 $\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{\infty - 50}{\infty + 50} = +1$

$v_{total} = v_{standing} + v_{incident} + v_{reflected} = v_{standing} + v_o + v_o \Gamma$

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At  $t = T_0$  wave arrives at the load end.  $v_L = v_{standing} + v_{incident} + v_{reflected} = 0 + 2.5 + 2.5(+1) = 5$  V. A +2.5 V wave front reflects back toward the source. After  $T_0$  another seconds it becomes incident on the source.

At  $t = 2T_0$   
 An important change of perspective. Now looking at the problem from the perspective of the source, not the loa.: The wave that was a *reflection from the load* (reflection: bouncing out of) is now considered an *incident wave* upon the source! (Incident: going into) The former reflection becomes incident on the source.  $v_S = v_{standing} + v_{incident} + v_{reflected} = 2.5 + 2.5 + 2.5(0) = 5$  V. This is the reflection from the source toward the load. Since "matched" there is no reflection. The dynamic behavior of the t-line is now in the past, the asymptotic solution has been reached.

15

**A practical example: T-line with matched source, open load**

$R_S = Z_C$   
 $Z_C = 50 \Omega$   
 $R_L = \infty$

$\Gamma_S = \frac{Z_S - Z_C}{Z_S + Z_C} = \frac{50 - 50}{50 + 50} = 0$   
 $\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{\infty - 50}{\infty + 50} = +1$

$v_{total} = v_{standing} + v_{incident} + v_{reflected} = v_{standing} + v_o + v_o \Gamma$

Assume it takes  $T_0$  seconds for a wave to traverse the transmission line one time in one direction, say from S to L.

- At  $t = 0^-$  the T-line is fully discharged. Thus  $v_{standing} = 0$ . At  $t = 0^+$   $v_S = 5 \frac{Z_C}{R_S + Z_C} = 2.5$  V. Wave starts propagating.
- At  $t = T_0$  wave arrives at the load end.  $v_L = v_{standing} + v_{incident} + v_{reflected} = 0 + 2.5 + 2.5(+1) = 5$  V. A +2.5 V wave front reflects back toward the source. After  $T_0$  seconds it becomes incident on the source.
- At  $t = 2T_0$  the reflection is incident on the source.  $v_S = v_{standing} + v_{incident} + v_{reflected} = 2.5 + 2.5 + 2.5(0) = 5$  V. Since there is no reflection from the source end, the problem ends here with the entire transmission line at 5 V.

Check: The asymptotic voltage everywhere on the t-line is  $(5 \text{ V}) \frac{Z_L}{Z_L + Z_S} = (5 \text{ V}) \frac{\infty}{\infty + 50} = 5 \text{ V}$ .

Asymptotic: The result after enough time—  
 Always the same as EGR 220 would give.

16



Same example, except with zero-ohm source, open load

$\Gamma_S = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{0 - 50}{0 + 50} = -1$ 
 $\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{\infty - 50}{\infty + 50} = +1$

$v_{total} = v_{standing} + v_{incident} + v_{reflected} = v_{standing} + v_o + v_o\Gamma$

Assume it takes  $T_0$  seconds for a wave to traverse the transmission line.

- At  $t = 0^+$  the T-line is fully discharged. Thus  $v_{standing} = 0$ . Also, at  $t = 0^+$   $v_S = 5 \frac{Z_C}{R_S + Z_C} = 5$  V. Wave starts propagating. *VOLTAJE DIVISION*
- At  $t = T_0$  wave arrives at the load end.  $v_L = v_{standing} + v_{incident} + v_{reflected} = 0 + 5 + 5(+1) = 10$  V. *!*  
A +5 V wave front reflects back toward the source. It becomes incident on the source.
- At  $t = 2T_0$  the reflection is incident on the source.  $v_S = v_{standing} + v_{incident} + v_{reflected} = 5 + 5 + 5(-1) = 5$  V.  
A -5 V wave front reflects back toward the load. It becomes incident on the load.
- At  $t = 3T_0$  wave arrives at the load end.  $v_L = v_{standing} + v_{incident} + v_{reflected} = 10 - 5 - 5(+1) = 0$  V.  
A -5 V wave front reflects back toward the source. It becomes incident on the source.
- At  $t = 4T_0$  wave arrives at the source end.  $v_S = v_{standing} + v_{incident} + v_{reflected} = 5 - 5 - 5(-1) = 5$  V.  
A +5 V wave front reflects back toward the load. It becomes incident on the source, etc. *forever 6% lossless*

17

Now, suppose the transmission line is not infinitely long, but is terminated at distance  $L$  with load  $R_L$ .

At the load end  $\frac{v_L(t)}{i_L(t)} = R_L$  and  $\frac{v_o(t)}{i_o(t)} = Z_C$  But in general,  $Z_C \neq R_L$  therefore...

If  $R_L > Z_C$  some of the incident current,  $i_o(x, t)$  will reflect back and  $v_L(t)$  will increase until ohm's law is satisfied for the load.  $v_o(L, t) + v_R(L, t) = i_L(t)R_L$ , but by KCL  $i_L(t) = i_o(L, t) - i_R(L, t)$ .

$$v_o(L, t) + v_R(L, t) = (i_o(L, t) - i_R(L, t))R_L$$

Substituting for the currents gives

$$v_o(L, t) + v_R(L, t) = (v_o(L, t) - v_R(L, t))(R_L/Z_C)$$

$$v_o \left( 1 - \frac{R_L}{Z_C} \right) = -v_R \left( 1 + \frac{R_L}{Z_C} \right)$$

$$v_R(R_L + Z_C) = v_o(R_L - Z_C)$$

$$v_R = v_o \left( \frac{R_L - Z_C}{R_L + Z_C} \right)$$

18

Now, suppose the transmission line is not infinitely long, but is terminated at distance  $L$  with load  $R_L$ .

$$v_R = v_o \left( \frac{R_L - Z_c}{R_L + Z_c} \right)$$

Define the reflection coefficient,  $\Gamma$   
 Also,  $R_L$  is usually generalized to  $Z_L$ . (So in our case,  $Z_L = R_L$ .)

$$\Gamma = \frac{Z_L - Z_c}{Z_L + Z_c}$$

$$v_R = v_o \Gamma$$

Finally suppose there was an initial charge on the transmission line, defined as  $v_{standing}$  and...  
 $v_o$  represents a change in voltage traveling down the transmission line. Then by superposition

$$v_L = v_{standing} + v_{incident} + v_{reflected} = v_{standing} + v_o + v_o \Gamma$$

19

Now, suppose the transmission line is not infinitely long, but is terminated at distance  $L$  with load  $R_L$ .

$$v_R = v_o \left( \frac{R_L - Z_c}{R_L + Z_c} \right)$$

Define the reflection coefficient,  $\Gamma$   
 Also,  $R_L$  is usually generalized to  $Z_L$ . (So in our case,  $Z_L = R_L$ .)

$$\Gamma = \frac{Z_L - Z_c}{Z_L + Z_c}$$

$$v_R = v_o \Gamma$$

Finally suppose there was an initial charge on the transmission line, defined as  $v_{standing}$  and  $v_o$  represents a change in voltage traveling down the transmission line. Then by superposition

$$v_L = v_{standing} + v_o + v_o \Gamma = v_{standing} + v_{incident} + v_{reflected}$$

20

Matched on all ends, tapped 1/3 of the way. Assume it takes  $T_0$  seconds to traverse 1/3 of the t-line or the tap

$R_S = Z_C = 50 \Omega$

$Z_C = 50 \Omega$

$R_{L2} = 50 \Omega$

$\Gamma_L = 0$

$R_{L1} = 50 \Omega$

$5u(t)$

$v_S(t)$

$v_T(t)$

$v_{L2}(t)$

$v_{L1}(t)$

$Z_C = 50 \Omega$

$Z_C = 50 \Omega$

$Z_C = 50 \Omega$

$\Gamma_S = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{50 - 50}{50 + 50} = 0$

$\Gamma_T = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{25 + 50}{25 + 50} = -\frac{1}{3}$

$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{50 - 50}{50 + 50} = 0$

Voltage divider Equation

$Z_L = 50 \parallel 50 = 25$

- At  $t = 0^-$  the T-line fully discharged. Thus  $v_{standing} = 0$ . Also, at  $t = 0^+$   $v_S = 5 \frac{Z_C}{R_S + Z_C} = 2.5$  V. Wave starts propagating.
- At  $t = T_0$  wave arrives at the tap.  $v_T = v_{standing} + v_{incident} + v_{reflected} = 0 + 2.5 + 2.5(-1/3) = 1.667$  V.  
A  $-5/6$  V =  $-0.833$  V wave front reflects back toward the source. Later it becomes incident on the source. Also, a 1.667 V wave is launched toward each load.
- At  $t = 2T_0$  the reflection is incident on the source.  $v_S = 2.5 - 0.833 - 0.833(0) = 1.667$  V. There is no reflection. Simultaneously the 1.667 V wave reaches the tap's load  $v_{L2} = 0 + 1.667 + 1.667(0) = 1.667$  V. No reflection.
- At  $t = 3T_0$  1.667 V wave arrives at the load end.  $v_L = 0 + 1.667 + 1.667(0) = 1.667$  V. No reflection.

Asymptotic voltage =  $5 \frac{25}{50+25} = 1.667$  (Check)

21

A final point: Reflections carry energy.  
They can do damage.  
In transmission lines they are caused by impedance mismatches.

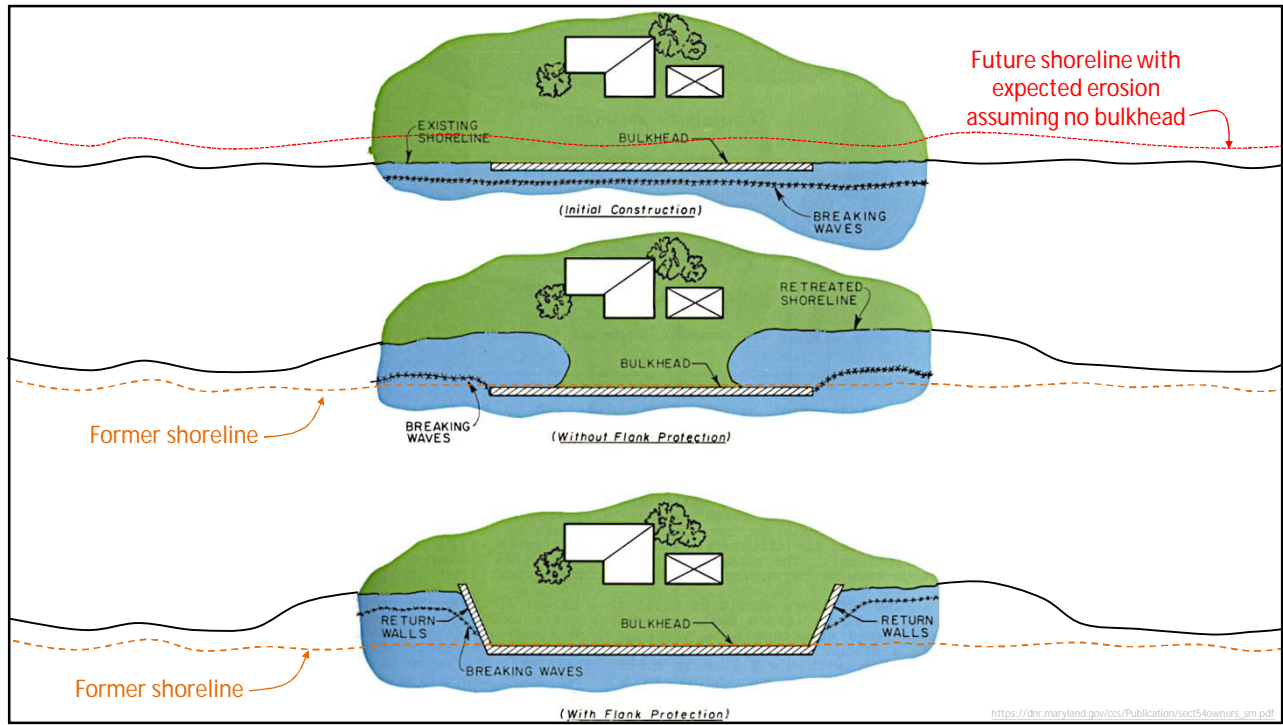
The rock riprap has no "give."  
That's an impedance mismatch.  
All water currents flowing up to the rock get reflected.  
Where does the energy go?

When the waves have died down (asymptotic solution is reached) the energy has been absorbed somewhere.

Running an output pin into a t-line having no load can overheat and melt the output driver in some unusual cases!

<https://slideplayer.com/slide/4476233/> at about 5:00

22



23